Intra-year Cash Flow Patterns: A Simple Solution for an Unnecessary Appraisal Error

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Introduction

The appraisal and academic communities have spent much time and effort in recent years developing and refining appraisal techniques to make them as theoretically correct and practically applicable as possible. As a result, income appraisal techniques such as discounted cash flow and capitalization of earnings are commonly used in the appraisal of income producing real property and closely held businesses. These techniques are theoretically sound and have become the primary valuation methods for many appraisers.

However, the high degree of difficulty of forecasting future revenues, expenses, profits and cash flows result in some unavoidable application problems. Actual results almost always deviate from forecasts to some degree because of unforeseeable events and conditions, changing relationships between costs and revenues, changes in government policy and other factors. Many of these problems are unavoidable and the appraiser's task is to limit errors as much as possible through analysis.

Whatever their theoretical soundness, there is one error built into most appraisal tools as they are commonly applied. This error concerns the intra-year timing of cash flows and returns. Appraisal techniques such as capitalization of earnings, Ellwood formulae and discounted cash flow as they are most often applied inherently assume that income or cash flows occur at the end of each year. This is obviously not realistic in the vast majority of cases. The resulting appraised values may be significantly in error because of this technical assumption implicit in the valuation tool. The appraisal process is difficult enough without having known errors built in an appraisal technique, especially if the errors are significant. This article discusses the problem and proposes a simple solution.

Assumptions Made in Income Appraisal Models

The essence of the income approach to appraisal is that the value of a property reflects the present value of future benefits of property ownership as measured by income or cash flows. The appraisal of an income producing property generally involves the use of an income valuation model in addition to market determined multiples of sales, earnings or cash flow and cost approaches. There are several income approaches which range from a simple net operating income (NOI) capitalization model to elaborate, comprehensive discounted cash flow models.

All income approaches require determination of three critical factors which determine the value of the property: return to the investor, timing of the return, and risk, which determines the required yield or discount rate. The definition of the return to the investor depends on the appraisal method employed. For example, the NOI capitalization method uses operating cash

flows prior to debt service. Appraisals based on cash flows to equity use cash flows after debt service.
The discount rate, also known as the yield rate, reflects the investor's required rate of return based on the risk and type of property interest being valued. This rate can be an all equity rate in which the returns to be discounted are after-tax cash flows to equity. Alternatively, in net operating income techniques the yield rate reflects returns to both debt and equity.

The third factor, timing of returns, takes into account when cash flows and profits are received and then adjusts to present value. The assumption that income or cash flow will be received at specific points in time is built into all of the appraisal methods and it is this assumption that is a common source of error in appraisals.

Some texts and articles include discussions of monthly or other cash flow patterns and some appraisers adjust for monthly cash flows (1, 487, 538)(2, 70). However, most sources and appraisals assume year end receipts. For example, The Appraisal of Real Estate contains an extensive discussion of how mortgage equity can be used to determine capitalization rates for income capitalization models (1, 521-554). All of the applications presented assume that the NOI is received at the end of each year. Other authors also present valuation models which usually specify annual returns and annual discounting (2, 415-429)(3, 210-245)(4, 73-84).

The reason that property returns are measured on an annual basis is likely due to convention and the fact that it is simply more convenient to accumulate and project data on an annual basis. In reality, however, cash flows almost never are received either at the beginning or at the end of the year but are spread out over the entire year. Many cash receipts, disbursements and debt service payments occur monthly due to normal billing procedures, payment schedules, loan contracts and other agreement.

For some properties, cash flows may be even more frequent. Business revenues normally are received daily while payroll and other cash expenses may be incurred weekly, biweekly or in other patterns. Other expenditures, such as taxes, may be paid quarterly or annually.

Realizable net benefits for investors, whether measured as cash flows to equity or some other measure, occur in the same pattern as the flow of receipts and expenditures. Thus monthly cash flows approximate reality much more closely than annual cash flows even though most appraisals assume yearly flows. Even if cash flows do not occur strictly in a monthly pattern, the monthly assumption will be accurate enough for use in almost all cases and is a substantial improvement over the assumption of year end receipts.

Adjustment for Monthly Cash Flows

Cash flows follow one of three patterns- annuities, perpetuities and uneven. Every appraisal using an income approach includes at least one of these and many include two or all three. Determining the present value of any of these involves an assumption as to when cash flows will be received within each year of the life of the appraised property. The problem of assuming year end rather than intra-year cash flows applies to each pattern.

Annuities

Although not a common situation, appraisals may involve the annuity cash flow patter in which the annual cash flows are essentially level over the project's life. For example, a project may have cash flows of $15,000 per year for five years:

End of:	Year 1	Year 2	Year 3	Year 4	Year 5
	$15,000	$15,000	$15,000	$15,000	$15,000

If end of year cash flows are assumed as they are in most printed present value tables, financial calculators and spreadsheets, the formula to value the annuity is: (1)

$$PVA(i_{nom}, N) = R\ X\ \frac{1 - \frac{1}{(1+i_{nom})^N}}{i_{nom}} = R\ X\ PVIFA(i_{nom}, N)$$

where:

$PVA(i_{nom},N)$	= present value of the annuity,
R	= annuity payment per year,
$PVIFA\ (i_{nom},N)$	= present value interest factor for man yearly annuity,
i_{nom}	= nominal annual interest (or discount) rate, and
N	= number of years over which the payments occur.

Assuming a required return of 12 percent results in a value of $54,072. However if the project really produces the cash flow monthly, the above formula misstates the pattern and value of the cash flows. The pattern is actually $1,250 per month for 60 months.

Month 1	Month 2	Month 3	Month 4	Month 60
$1,250	$1,250	$1,250	$1,250	$1,250

To find the present value of this stream, a monthly discount rate must be applied. An annual effective interest rate of 12 percent compounded annually does not translate to a monthly rate of 1 percent. The formula used to find the monthly compounded rate implicit in a nominal annual rate is: (2)

$$i_m = (1 + i_{nom})^{1/12} - 1$$

where:

i_m = effective monthly interest rate.

Thus the monthly interest rate implicit in an annual rate of 12 percent is: (3)

$$i_m = (1.12)^{1/12} - 1 = 0.0094888$$

The annuity formula for monthly cash flows is: (4)

$$PVIFA(i_m,12N) = \frac{1 - \frac{1}{(1+i_m)^{12N}}}{i_m}$$

where:
$PVIFA(i_m,12N)$= present value interest factor for monthly annuity.

Applying this monthly rate to the above example results in a present value of $56,985 as opposed to the $54,072 value calculated based on annual flows. Thus, an appraiser using this technique with annual cash flows and an annual yield rate has a built-in error in the final value of 5.4 percent. This error is due solely to the mathematical assumptions implicit in the present value equation and not to normal forecasting difficulties.

In many cases, appraisers use printed present value tables that assume annual cash flows. A table of adjustment factors can be derived to adjust these tables to the monthly cash flow assumption using the following equation: (5)

$$AdjustmentFactor = \frac{PVIFA(i_m,12N)}{PVIFA(i_{nom},N)}$$

The adjustment factors vary with interest rates but not with the number of years and thus only a single adjustment factor is need for each interest rate. The adjustment factors for annual interest rates varying from 8 percent to 30 percent appear in Table 1.

Table 1
Annual Monthly Adjustment Factors

Annual Interest Rate	Adjustment Factor	Annual Interest Rate	Adjustment Factor
1%	1.004575	16%	1.071369
2	1.009134	17	1.075709
3	1.013677	18	1.080036
4	1.018204	19	1.084350
5	1.022715	20	1.088651
6	1.027211	21	1.092939
7	1.031691	22	1.097215
8	1.036157	23	1.101479
9	1.040608	24	1.105731
10	1.045045	25	1.109971
11	1.049467	26	1.114198
12	1.053875	27	1.118415
13	1.058269	28	1.122619
14	1.062649	29	1.126812
15	1.067016	30	1.130994

The example given above can be used to illustrate the use of the table. A present value of $54,072 was calculated for the five year stream of annual cash flows of $15,000 using a yield rate of 12

percent. The adjustment factor in Table 1 for a 12 percent return is 1.053875. Multiplying this factor by $54,072 results in an adjusted present value of $56,985, which is the same as the present value calculated above using monthly cash flows.

For appraisers using financial calculators, spreadsheets or computerized appraisal software packages with annual cash flows, the adjustment is the same. Simply multiply the final value by the adjustment factor in Table 1 for the appropriate interest rate. Alternatively, the problem can be solved by using monthly cash flows and the monthly interest rate derived from Equation (2) in the calculator, spreadsheet or software, if the software will allow it.

Perpetuities

A more common pattern of cash flows assumed in appraisals is a perpetuity which is used in cases in which the stream of benefits is expected to be level each year and continue forever or its practical equivalent. This pattern is the basis for the widely used capitalization methods of NOI, net income and cash flows. A perpetuity assuming year end flows would appear as:

End of:	Year 1	Year 2	Year 3	infinity
	$15,000	$15,000	$15,000	$15,000

The formula for calculating the present value of this pattern is: (6)

$$\text{Present Value} = \frac{\text{Net Operating Income (NOI)}}{\text{Overall Capitalization Rate}}$$

Assuming a capitalization rate of 12 percent, the present value of the above perpetuity is $125,000. However, if monthly cash flows are used, the pattern is actually:

End of Month	Year 1		Year 2		Infinity
	January	February	January	February	
	$1,250	$1,250	$1,250	$1,250	$1,250

Using a monthly capitalization rate of 0.094888 percent as calculated in Equation (2), the present value is actually $131,734 from the monthly adaptation of Equation (6): (6B)

$$\text{Present Value} = \frac{\$1250}{0.0094888} = \$131{,}734$$

The factors from Table 1 also can be used to adjust perpetuities from year end to monthly cash flows. In this case, the value of $125,000 multiplied by the 12 percent adjustment factor of 1.053875 yields the same adjusted value of $131,734 calculated above.

Uneven Cash Flows

In many cases, cash flows vary from period to period. In these situations, each cash flow is treated independently and the appraiser values a series of individual flows using the following equation for each: (7)

$$\text{Pr}\,esentValue_{\,T} = \frac{CF_T}{(1+i_{nom})^t} = CF_T\,X\,\frac{1}{(1+i_{nom})^T} = CF_T\,XPVIF\$_{(inom,T)}$$

where:
Present $Value_T$ = present value of Year T's cash flow,
CF_T = year end cash flow in Year T, and
$PVIF\$_{(i\ nom,T)}$ = present value interest factor for year T's cash flow.

For example, assume an income producing property is projected to have the following annual, end of year cash flows:

Year 1	Year 2	Year 3
$9,000	$12,000	$18,000

Using the same required yield rate of 12 percent and Equation (7), the present value of this cash flow stream is $30,414. If the additional assumption is made that these cash flows occur monthly, an adjustment identical to those above can be made. The $30,414 present value is multiplied by the 1.053875 factor for 12 percent to arrive at an adjusted value of $32,053.

The accuracy of this adjustment can be proven by calculating present values of each year's cash flow separately using the monthly receipt assumption. As an illustration, consider the cash flows in Year 3 of $1,500 per month. To find the value today of these cash flows, two steps must be completed. First, the cash flows must be discounted to the beginning of Year 3 using the annuity factor for twelve periods with the monthly interest rate of 0.94888 percent as calculated above using Equation (4). The resulting amount is then discounted to the present for two years using the annual interest rate of 12 percent. The two year discounting period is used because of the fact that discounting the monthly flows establishes the value as of the beginning of Year 3 which is the same as the end of Year 2. This point is two years in the future from today. The process is illustrated as:

Step 1: Discount Year 3's monthly cash flows to the beginning of the year:

Year 3:

January	February	March	. . . . .	November	December
$1,500	$1,500	$1,500	$1,500	$1,500	$1,500

$16,937
Value at beginning of Year 3 (end of Year 2)

Step 2: Discount this value to today:

$16,937
Value at beginning of Year 3 (end of Year 2)

Value Today
$13,502

The process is repeated for each year's cash flow and the values for each year are summed to arrive at final total value. Mathematically, the calculation is:

This final value is the same as calculated above using the adjustment factor from Table 1.

Year	Monthly Cash Flow	0.94888% Monthly Present Value Annuity Factor 12 months	Present Value at Beginning of Year	12% Present Value Factor Present Value	
1	$750	11.2915	$8,469	1.0000	$8,469
2	1,000	11.2915	11,292	0.8929	10,082
3	1,500	11.2915	16,937	0.7972	13,502
				Final Value:	$32,053

Reversion Values

Many appraisal techniques, especially discounted cash flow and discounted earnings, involve forecasting individual uneven annual returns for a number of years and capitalizing the perpetuity at the end of this time to establish a terminal or reversion value. If annual returns are used in this process, the same timing error occurs. Thus, reversion values should be adjusted in the same manner as annual cash flows.

Mid-Year Cash Flows

A simple and approximately correct alternative to the monthly adjustment is to assume that all cash flows occur at mid-year. Under this assumption, the adjustment factor for all present values becomes: (8)

$$ApproximateAdjustmentFactor = \sqrt{1 + i_{nom}}$$

For example, using this equation for 12 percent the adjustment factor is 1.058301 compared to the factor of 1.053875 in Table 1. The resulting values would be approximately 0.4 percent different. The mid-year factor for 20 percent is 1.095445 compared to 1.088651 shown in Table 1 or a difference of 0.6 percent.

This observation raises two important points. First, while these differences are relatively small, a built-in error of approximately 0.5 percent is large enough that many appraisers may want to use the adjustments in Table 1 in the interest of accuracy. Secondly, the closeness of the mid-year adjustment to the more accurate factors presented in Table 1 and their substantial difference from year end factors reinforce the position that an adjustment for intra-year cash flow patterns should be made in order to avoid errors.

Summary

As they are commonly described in the literature and applied in practice, appraisal techniques have a built-in conceptual error. This error occurs because of the clearly unrealistic assumption that annual cash flows occur at the end of the respective years of a property's or business's life. This article discusses the problem as it relates to the various cash flow patterns encountered in

income property and closely held business appraisals and develops a theoretically sound, simple adjustment to correct it.

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